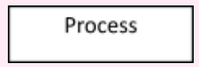
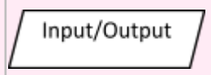
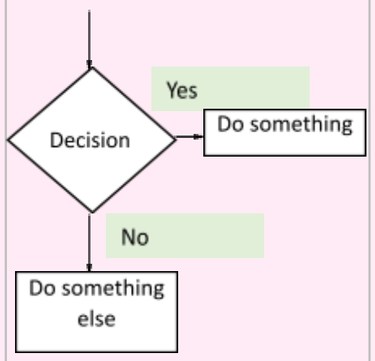
Writing and Interpreting Algorithms

Start and Stop 	**Process – An operation that the algorithm performs** 
Connector – Links all the other symbols together 	**Input and Output of data that is read in and written out** 

Decision is the same as a selection (if then … else)
 
	`IF answer is "yes" THEN`
	`  do something`
	`ELSE IF answer is "no"`
	`  do something else`
	`ENDIF`

We can represent algorithms using flowcharts

Pseudocode

We can represent algorithms using pseudocode

	Example	Python equivalent
Variable assignment	a ← 10	`a = 10`
Constant assignment	constant PI ← 3.142	`PI = 3.142`
Input	a ← USERINPUT	`a = input()`
Output	OUTPUT "Bye"	`print("Bye")`

Arithmetic Operators
Add
Multiply
Divide
Subtract
Integer division
Modulus (remainder)
	`+`
	`*`
	`/`
	`-`
	`a ← 7 DIV 2`
	`a ← 7 MOD 2`
		`+`
		`*`
		`/`
		`-`
		`a= 7 // 2`
		`a = 7 % 2`

Relational Operators		

Less than
Greater than
Equal to
Not equal to
Less than or equal to
Greater than or equal to
	`<`
	`>`
	`=`
	`≠ or <>`
	`≤`
	`≥`
		`<`
		`>`
		`==`
		`!=`
		`<=`
		`>=`

Boolean Operators
AND
OR
NOT
	`AND`
	`OR`
	`NOT`
		`AND`
		`OR`
		`NOT`

Selection
if ..
	`IF i > 2 THEN`
	`  j ← 10`
	`ENDIF`
		`if i  > 2:`
		`  j=10`

if .. else …	`IF i > 2  THEN`
	`  j ← 10`
	`ELSE`
	`  j ← 3`
	`ENDIF`
		`if i > 2:`
		`  j=10`
		`else:`
		`  j=3`

if … else if … else	`IF i ==2 THEN`
	`  j ← 10`
	`ELSE IF i==3`
	`THEN`
	`  j ← 3`
	`ELSE`
	`  j ← 1`
	`ENDIF`
		`if i ==2:`
		`  j=10`
		`elif i==3:`
		`  j=3`
		`else:`
		`  j=1`

Iteration
While loops
	`a ← 1`
	`WHILE a < 4`
	`  OUTPUT a`
	`  a ← a + 1`
	`ENDWHILE`
		`while a<4:`
		`  print(a)`
		`  a=a+1`

For loops	`FOR a ← 0 TO 3`
	`  OUTPUT a`
	`ENDFOR`
	`a ← 1`
		`for a in`
		`range(3):`
		`  print(a)`

Repeat loops	`REPEAT`
	`  OUTPUT a`
	`  a ← a + 1`
	`UNTIL a←4`	

Arrays

	Example	Python equivalent
Set up array	`a ← [1,2,3,4,5]`	`a=[1,2,3,4,5]`
Access element	`a[0]`	`a[0]`
Update element	`a[0] ← 4`	`a[0] = 4`
Set up 2D array	`a ← [[1,2],[3,4]]`	`a = [[1,2],[3,4]]`
Access 2D element	`a[0][1]`	`a[0][1]`
Update 2D element	`a[0][1] ← 4`	`a[0][1] = 4`

Subroutines
procedure
	`SUB hello()`
	`  OUTPUT "hello"`
	`ENDSUB`
		`def hello():`
		`  print("hello")`

Function (with paramerters and return)	`SUB add(n)`
	`  a ← 0`
	`  FOR a ← 0 TO n`
	`  a ← a + n`
	`  ENDFOR`
	`  RETURN a`
	`ENDSUB`
		`def add(n):`
		`  a=0`
		`  for a in range(n+1):`
		`  a=a+n`
		`  return a`

Built-in functions Length of array	LEN(a)	len(a)
Random integer	RANDOM_INT(0, 9)	import random random.randint(0,9)

Abstraction

Representational abstraction
Abstraction allows us to remove unnecessary detail from a problem leaving only the essential features thereby making it easier to solve. Maps are examples of representational abstraction.

Abstract generalisation
With abstract generalisation we identify common (general) characteristics thereby enabling us to group similar constructs together into a hierarchy

Abstract generalisation is also the ability to see patterns so that we can recognise problems or parts of problems that we may have solved before. Lots of programming is concerned with reusing pieces of code that were originally developed for other solutions. Even within a piece of code we are writing we may notice that quite a lot of our code is repeated. It is our ability to notice those repetitions that help us write more succinct and generalisable code using functions perhaps.

Procedural abstraction
Abstract away the actual values used in a computational method. In that sense algebra and formulas are abstractions. The following expressions have the same form:

```
(1+2) x 3
(7+9) x 2
(5.5 + 12.3) x 18.1
```

We can abstract them away algebraically as:

```
(a+b) x c
```

Functional abstraction
- A functional abstraction maps an input to an output. The function returns a value given a certain input.
- Functional abstraction is an extension of procedural abstraction. A procedural abstraction might form part of a functional abstraction.

```
def calc(a,b,c):
    return (a+b)*c
print(calc(1,2,3))
```

- In programming we abstract details using functions. We do not need to know how functions work to use them. The functions themselves can be a black box to us. The details of how the function works have been abstracted away.

Data abstraction
The details of how the data are represented are hidden. We do not need to worry how ASCII characters, real numbers and integers are represented. Real numbers can be represented using exponent and mantissa in binary, but we do not need to concern ourselves about this when we a writing programs. We can have more complex abstract data types. These include queues, stacks graphs, trees, hash tables, dictionaries and vectors.

Problem abstraction
Remove details of a problem until you are left with a problem that you already know how to solve. This allows us to use solutions that have been applied to analogous problems. For instance Euler solved the Konigsberg bridge problem (is it possible to cross all bridges only once) by reducing it down to a graph problem, that he already knew how to solve.

Information hiding
In OOP, this is where data that do not contribute to the essential characteristics of an object are hidden. These attributes and methods are private and not accessible from outside an object. Essential characteristics of the object can be accessed via an interface. Also we do not need to concern ourselves with local variables in functions.

Decomposition
Decomposition is the breaking down of a complex problem into smaller more manageable problems that are easier to solve. Each component of the program completes a specific task. This allows algorithms to be more modular and therefore more intuitive.

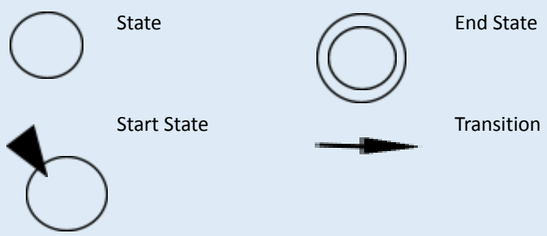
Composition
Composition is combining the procedures together to form compound procedures in order to solve a greater part of the problem that each of the procedures can solve separately. Specifying the interface between the components is important otherwise they would not fit together.

Automation
- Putting models into action using algorithms
- Putting the abstractions into algorithms and putting the algorithms into code.
- Developing computer models that concentrate on the essence of a problem. The models are a simplified representation of reality where assumptions are made.
- For instance, weather forecast models use mathematical models and physics to model the atmosphere as a fluid, which is a good way to run simulations and predict the weather up to a few days ahead.
- It is not possible to model the billions of variables, so simplifications are made to help solve the problem. As computers get more powerful and algorithms improve and we have more data we get better at predicting the weather.

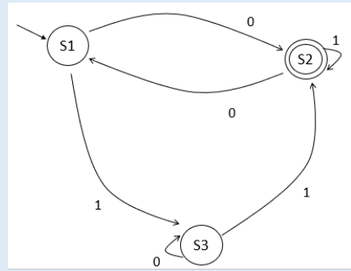
Finite State Machines (FSMs)

FSMs are a model of computation that allow us to understand how computers work. FSMs consist of a set number of states that allows the transition between states and are determined by a fixed set of inputs and have a set of outputs.

Notation for FSM



Finite state diagrams are a graphical way of presenting finite state machines.



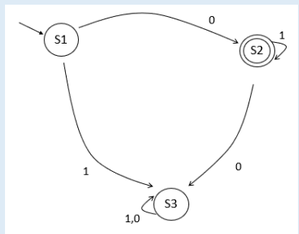
- S1, S2 and S3 are the states
- Each transition edge has an input value
- S1 is the Start State
- S2 is the Accept State)
- For the input sequence to be valid the sequence must end on the accept state (S2)

Example sequences
- 0 0 0 1 1 - Valid
- 1 0 0 1 - Valid
- 1 0 1 0 - Invalid
- 1 0 1 - Valid

State transition tables are another way of representing FSM.

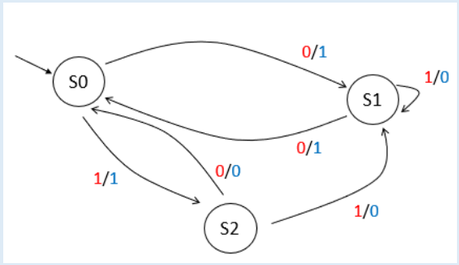
Start state	Input	New State
S1	0	S2
S1	1	S3
S2	0	S1
S2	1	S2
S3	0	S3
S3	1	S2

Trapping invalid input – In the following example S3 captures invalid input there is no what to transition to another state once S3 has been achieved.



Mealy Machines
- The FSM we have looked at so far have a valid and invalid state. The valid state is the accept state
- Mealy machines are a type of FSM that have outputs on each transition and have no end state

Example Mealy Machine



The red number is the input and the blue number is the output.

Input	Output
0 1 1 1 0 1 1 0 1	1 0 0 0 1 1 0 1 1
1 1 1 0 0 1 0 0 0	1 0 0 1 1 1 1 1 1
1 0 1 0 0 1 1 1 0	1 0 0 1 1 0 0 0 1

Corresponding state transition diagram

Start state	Input	New State	Output
S0	0	S1	1
S0	1	S2	1
S1	0	S0	1
S1	1	S1	0
S2	1	S1	0
S2	0	S0	0

Turing Machines

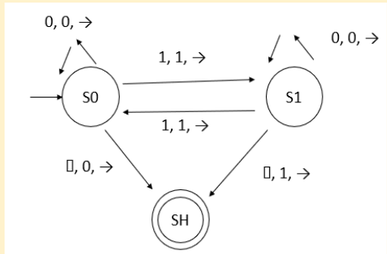
Purpose of Turing Machines

- Turing machines are a model of computation that help us understand how algorithms can be solved computationally.
- If a problem is computable then it can be solved by a Turing machine (Church-Turing thesis).
- Turing machines can be used determine whether an algorithm is computable.

How a Turing Machine Works

- A Turing machine is a finite state machine with a tape of infinite length that is divided into squares. This is the memory of the machine.
- Has a finite set of symbols, commonly 0 , 1 and • which indicates no value. Each square on the tape takes on one of the values of the symbols.
- Has a head which can read and write to the tape and move along the tape in either direction.
- Has a finite set of states. It can have a start state and must have a halting state.
- Behave as interpreters because the deal with one instruction at a time.
- Turing machines can be expressed using:
  - Finite State Machines / diagrams
  - State transition tables
  - State transition functions

Finite state machine for even parity generator



First value is the symbol read, Second value is the symbol to write and third value is the direction in which to move the head

State transition table for even parity generator

State	Read	Write	Move	Next state
S0	0	0	→	S0
S0	1	1	→	S1
S0	•	0	→	SH
S1	0	0	→	S1
S1	1	1	→	S0
S1	•	1	→	SH

State transition function for even parity generator

∂(current state, input symbol) = (next state , output symbol ,direction)

∂(S0,0) = (S0,0,→)
∂(S0,1) = (S1,1,→)
∂(S0,•) = (SH,0,→)
∂(S1,0) = (S1,0,→)
∂(S1,1) = (S0,1,→)
∂(S1,•) = (SH,1,→)

Worked example:

Tape used for even parity generator	• 1 0 1 1 1 0 • • • *The green arrow denotes the position of the read/ write head*
Step 1	• 1 0 1 1 1 0 • • • S0
Step 2	• 1 0 1 1 1 0 • • • S1
Step 3	• 1 0 1 1 1 0 • • • S1
Step 4	• 1 0 1 1 1 0 • • • S0
Step 5	• 1 0 1 1 1 0 • • • S1
Step 6	• 1 0 1 1 1 0 • • • S0
Step 7	• 1 0 1 1 1 0 • • • S0
Step 8	• 1 0 1 1 1 0 0 • • SH

Universal Turing Machines

- For each operation a different Turing machine has to be created, so this is not ideal.
- For a Turing machine, the state transition diagram / function / FSM are the instructions so is separate from the tape
- A Universal Turing machine is a Turing machine that can execute another Turing machine. The instructions of the Turing machine are stored on the tape.
- This model of computing is what is used in modern computers today where both the program instructions and the data are stored in memory.